Journal of Applied Mechanics and Technical Physics, Vol. 43, No. 2, pp. 245–248, 2002

OCCURRENCE OF TORNADO-LIKE VORTICES IN A ROTATING FLUID UNDER FORCED INERTIAL OSCILLATIONS OF LARGE AMPLITUDE

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UDC 532.5

Experimental estimates are obtained for the main parameters of tornado-like vortices that arise from excitation of forced axisymmetric inertial oscillations of large amplitude in a rigidly rotating fluid.

The property of hyroscopic elasticity of a rigidly rotating fluid is well known and has been fully described in the literature. A rotating fluid is usually regarded as a medium capable of transmitting and sustaining oscillatory motions with frequencies from the inertial range [1]. At small vibration amplitudes, this agrees with experimental data. However, when the oscillation amplitude becomes large enough, the flow pattern changes cardinally under the action of these oscillations, which, in turn, changes radically the structure and parameters of the rotating fluid flow.

Makarenko and Tarasov [2, 3] established that intense oscillating (with period of forced oscillations) tornado-like vortices arise when large-amplitude forced inertial oscillations of resonant frequencies are excited in a fluid which fills a cylindrical tank rotating about its axis. In particular, excitation of axisymmetric oscillations gives rise to an oscillating tornado-like vortex (near the cylinder axis), in whose core the maximum vorticity amplitude is $\omega_z \simeq 50\omega_0$ (ω_0 is the vorticity of the unperturbed fluid which rotates at an angular velocity $\Omega = \omega_0/2$). At this time, the radius of the core (radius of the circle on which the azimuthal velocity component reaches a maximum value) is $r_0 \simeq 0.05R$ (R is the radius of the rotating cylinder). The vorticity in the vortex core changes in time from the initial value ω_0 to the maximum but the direction of rotation of the fluid (sign of ω_z) does not change.

The goal of the present work is to confirm (verify) the existence of the indicated phenomenon on a setup with different parameters and to obtain further information on the effect of experimental conditions and the method of exciting forced oscillations on the main characteristics of tornado-like vortices.

Experimental Setup. The experimental setup is a transparent cylindrical tank whose vertical axis rotates at a given angular velocity. The bottom of the tank is made of a thin deformable rubber film, and its upper butt-end is sealed with a rigid transparent cover. The inner radius of the tank is $R = 25$ cm, and its height is $H = 38.6$ cm. The tank was completely filled with a fluid (water). Compressed air was injected into the cavity under the rubber bottom of the tank to prevent deflection of the bottom under water. Under the rubber bottom of the tank, a special plunger of a spherical shape (chord length 12 cm and segment height 1.2 cm) made of fluoroplastic performed vertical oscillations, deforming the bottom axisymmetrically with specified amplitude and frequency.

For small-amplitude inertial oscillations in a rotating fluid placed in a cylinder of specified dimensions, a complete set of natural modes is known and any of the eigenfrequencies can be calculated. This allows one to choose input parameters of the experiment and to imagine the flow pattern beforehand because in the given configuration, the eigenfrequencies calculated by linear theory are in good agreement with the recorded ones [1]. In particular, for the fundamental mode of axisymmetric oscillations (one maximum of the modulus of the stream function in meridional section of the cylinder) used in the experiment, the fundamental frequency is

$$\omega = 2\Omega/\sqrt{1 + \mu^2 H^2/(\pi^2 R^2)},$$

where Ω is the angular velocity of rotation of the cylinder and $\mu = 3.83$ is the first root of the equation $J_1(\mu) = 0$ (J_1 is a Bessel function of the first kind).

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Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 43, No. 2, pp. 87–91, March–April, 2002. Original article submitted December 24, 2001.

0021-8944/02/4302-0245 $27.00

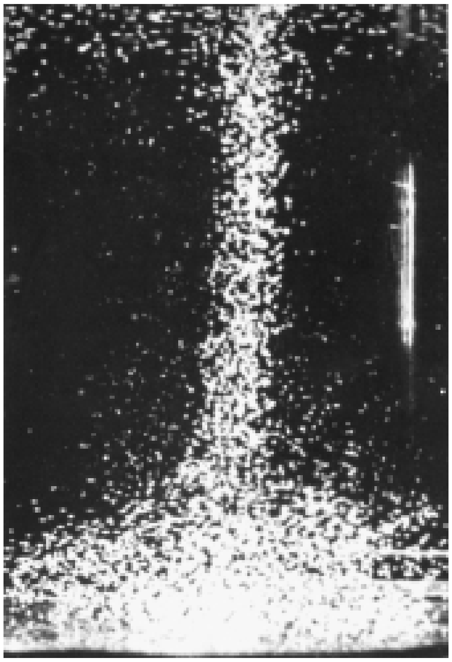

Fig. 1

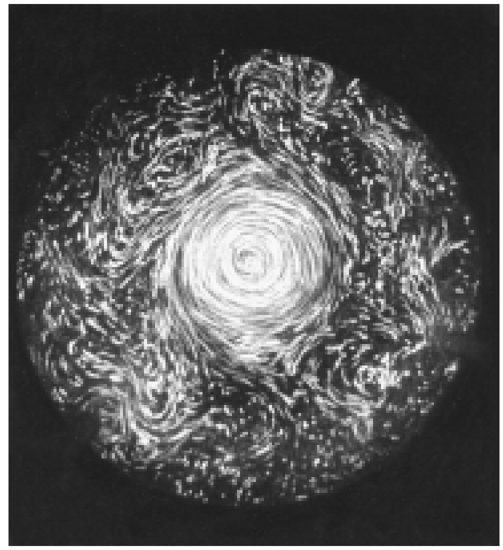

Fig. 2

When the frequency of the plunger coincides with the resonant frequency of the fluid which is rigidly rotating at the initial time, a concentrated tornado-like vortex is formed in the fluid along the axis of the tank. Visualization of this vortex was performed using fine polystyrene spheres which were poured in the tank before the beginning of the experiments. The density of the spheres was close to the fluid density in the tank. The vortices that arose in the fluid were recorded by a camera from the transparent butt-end of the tank or from its lateral surface area.

Photographs of the concentrated vortex formed along the axis of the rotating tank, taken from the lateral surface and from the butt-end of the tank are given in Figs. 1 and 2, respectively.

Measurement Data. Experiments were performed for fixed values of the rotation frequencies of the cylinder and oscillations of the plunger. In the measurements, the rotation period of the tank was $T_0 = 2.4$ sec (initial vorticity of the fluid $\omega_0 = 5.24$ sec^{-1}); the oscillation period of the plunger $T = 2.55$ sec coincides with the period of the fundamental eigenfrequency of axisymmetric inertial oscillations of the rigidly rotating fluid in the cylinder; the amplitude of vertical displacements of the plunger was $h = 3$ cm. In this regime, an oscillating, concentrated, vertical vortex arose in the tank, which was not stationary in space (moved in the tank in the neighborhood of the cylinder axis) and in time. Periodic changes of the azimuthal velocity in the vortex and its horizontal dimensions (with forced frequency) were observed.

To center the vortex after establishment of the oscillatory regime, we stopped the rotation of the cylinder, and at an appropriate moment within 10–15 sec after the stopping, determined the velocity field. in this time interval, the amplitude of the inertial oscillations practically did not change (typical decay time for the setup used is $\tau \sim H/\sqrt{\nu\omega_0} \sim 10^3$ sec). It is assumed in this case that the nonstationary Ekman boundary layer, in which the flow is responsible for centering of the vortex, does not significantly affect the main parameters of the tornado-like vortex (boundary-layer width $\sqrt{\nu/\omega_0} \sim 0.1$ cm). Nevertheless, one might expect the occurrence of some singularities near the butt-ends, such as vortices of small radius, which arise from separation of the boundary layer in the neighborhood of the axis. However, without using this expedient it is not possible to measure the velocity field and obtain significant information.

The velocity field in the vortex was determined in the phase of its maximum compression (this moment was determined visually) in a given horizontal section of the tank by recording tracks of the particles moving together with the fluid. The particles in this section were visualized by a flat beam of light 1 cm wide, which was placed at various heights ($H_1 = 14$ cm, $H_2 = 19.3$ cm, and $H_3 = 37$ cm) from the bottom of the tank. A pattern of intermittent particle tracks was obtained by means of a rotating obturator, which overlapped the camera lens periodically at a frequency of 120 Hz. As a result, three positions of each moving particle were recorded in one

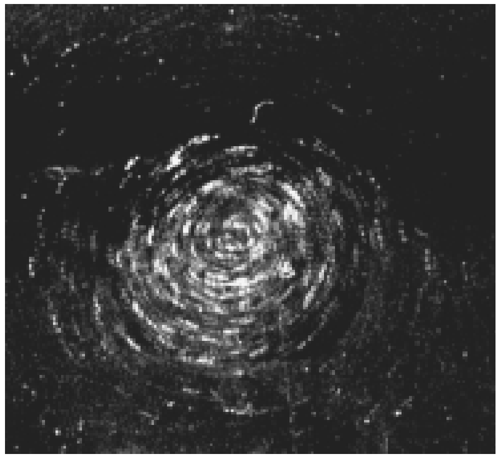

Fig. 3

photograph. One of such photographs is given in Fig. 3. Processing of the photographs yielded the azimuthal velocity component V at various distances from the center of the vortex r in the selected horizontal section.

Results of the processing are given in Figs. 4–6. The azimuthal velocity is shown by points for a fixed coordinate system and by triangles for a system rotating together with the cylinder. For vortices of low intensity (maximum vorticity $\omega_z < 10\omega_0$) there may be no maximum of the azimuthal velocity in the neighborhood of the cylinder axis in the fixed coordinate system. In the rotating system, such a maximum is always present and the value of the radius on which it is reached determines the degree of concentration of the vortex (radius of the vortex core). For vortices of high intensity (maximum vorticity $\omega_z \gg \omega_0$) the difference between the fixed and rotating systems becomes insignificant. The slope of the straight lines determines the magnitude of vorticity in the relevant systems (solid lines refer to the fixed system, and dashed lines refer to the rotating system). From Fig. 4 ($H_1 = 14$ cm) it follows that the radius of the vortex core is $r_0 \simeq 3.5$ cm. In this case, the maximum azimuthal velocity (relative to the fixed system) is $V \simeq 35$ cm/sec (maximal azimuthal velocity of the rigidly rotating fluid at the initial time $V_0 = 65.4$ cm/sec), and the maximum vorticity in the vortex core is $\omega_z \simeq 3.8\omega_0$. In Fig. 5 ($H_2 = 19.3$ cm), the value of $V \simeq 45$ cm/sec is reached at $r_0 \simeq 3.5$ cm, and the maximal vorticity is $\omega_z \simeq 4.9\omega_0$. In Fig. 6 ($H_3 = 37$ cm), the value of $V \simeq 45$ cm/sec is reached at $r_0 \simeq 3$ cm, and the maximum vorticity is $\omega_z \simeq 5.7\omega_0$. In Fig. 6, the second solid straight line with a larger slope (in Fig. 6, dashed curves were not drawn) is drawn through a group of points that departed markedly from the positions of most of the remaining points. Possibly, the occurrence of these values of the azimuthal velocity suggests the formation (due to separation of the boundary layer from the butt-end surface of the cylinder) a secondary near-axis vortex (which is coaxial with the main vortex but have smaller radius) whose vorticity is greater than that of the basic tornado-like vortex.

The considerable scatter in the experimental points is explained by the fact that the flow was not strictly axisymmetric and the vortex axis did not coincide wit the cylinder axis, which was due to the nonstationary position of the vortex in space. Even after centering, its vertical axis drifted in an irregular fashion relative to the center of the tank. In addition, the formation of lateral vortices weaker than the axial vortex is possible, which cannot be controlled under the present experimental conditions. It is not improbable that quasiturbulent velocity pulsations make a significant contribution to the scatter in velocity values (developed turbulence in the vortex was not observed) because the Reynolds number, which can be taken as $\mathrm{Re} = H^2/(\nu T) \sim 10^4$, is large enough.

Vertical oscillatory displacements of neutrally buoyant particles along the axis of the vortex were not measured. The observed amplitudes of displacement of the particles were comparable with the height of the tank. From this, it is possible to estimate the order of magnitude of the vertical velocity on the vortex axis $W \sim H/T \simeq 15$ cm/sec. The radii of orbits of particle motion around the vortex axis also changed quasiperiodically, deviating about a factor of 1.5–2 from the radii in the unperturbed positions. The irregularity of these oscillations does not permit more accurate estimates of this quantity to be obtained.

The experiments performed showed that on the axis of a cylindrical tank there is no constant vorticity component much larger than ω_0.

The results confirm qualitatively the conclusion of [2, 3] on the occurrence of a tornado-like vortex during excitation of large-amplitude inertial oscillations in a rotating fluid. However, its intensity turned out to be much

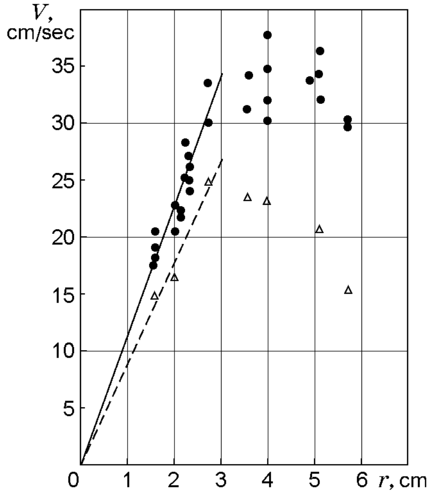

Fig. 4

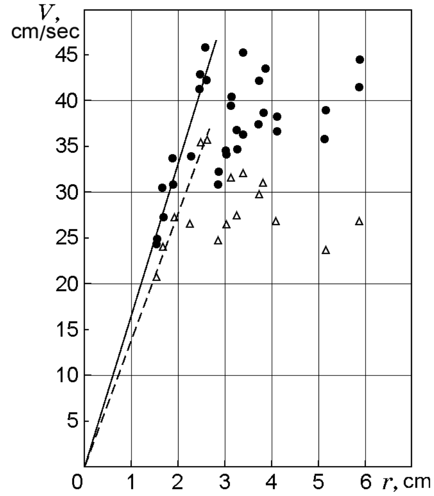

Fig. 5

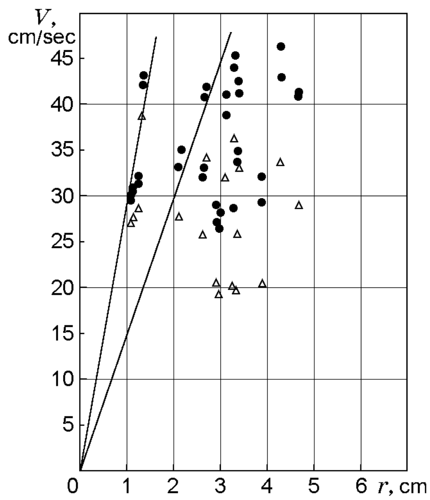

Fig. 6

lower than that in [2, 3]. This is apparently due to the fact that despite the use of the resonant frequency, the intensity and dimensions of the vortex depend significantly on the dimensions of the setup, the method of exciting oscillations, and the amplitude and type of deformation of the butt-end at the bottom of the cylinder that introduces perturbations. For example, in [2, 3], the ratio is h/H almost twice and the ratio H/R is half that in the given experiment. The other parameters differ as well. This circumstance impedes a comparison of the results obtained on different setups and requires systematic (rather labor-consuming) experiments to determine the effect of parameters of the setup and the method of exciting large-amplitude forced inertial oscillations on the intensity of the vortex that arises. It is possible that there is an optimum set of parameters that leads to the formation of an oscillating tornado-like vortex of maximum intensity.

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 99-01-00597).

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